Abstractions of Dynamical Systems

Colas Le Guernic

October 28, 2010
Motivations

A typical example:

- a differential equation $\dot{x} = f(x)$, $f : \mathbb{R}^d \rightarrow \mathbb{R}^d$
- an initial point $x_0$
- a set of “bad” states $F$
A typical example:

- A differential equation \( \dot{x} = f(x) \), \( f : \mathbb{R}^d \rightarrow \mathbb{R}^d \)
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- a differential equation $\dot{x} = f(x)$, $f : \mathbb{R}^d \rightarrow \mathbb{R}^d$
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A typical example:

- a differential inclusion $\dot{x} \in f(x), \quad f : \mathbb{R}^d \to \mathcal{P}(\mathbb{R}^d)$
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A typical example:

- a differential inclusion \( \dot{x} \in f(x), \ f : \mathbb{R}^d \rightarrow \mathcal{P}(\mathbb{R}^d) \)
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Hybrid Systems

Outline

Introduction
Motivations
Hybrid Systems
State of the Art
Abstraction
Conclusion

\[ \dot{x} \in f_1(x) \]
\[ x \in G_{1,2} \]
\[ x \leftarrow R_{1,2}(x) \]

\[ \dot{x} \in f_2(x) \]
\[ x \in G_{2,3} \]
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\[ \dot{x} \in f_3(x) \]
\[ x \in G_{3,1} \]
\[ x \leftarrow R_{3,1}(x) \]

\[ \dot{x} \in f_4(x) \]
\[ x \in G_{2,4} \]
\[ x \leftarrow R_{2,4}(x) \]
A few reflexions on:

- Reachability for some specific classes of functions $f$.
- Abstractions of arbitrary systems using these specific functions.

Including some ongoing work:

- On Linear Parameter Varying systems with Matthias Althoff and Bruce Krogh.
State of the Art

$f : \mathbb{R}^0 \rightarrow \mathcal{P} \left( \mathbb{R}^0 \right)$

$f(x) = \{1\}$

$f(x) = \mathcal{P}$

$f(x) = A\{x\} \oplus \mathcal{U}$

$f(x) = A\{x\}$

$f : \mathbb{R}^d \rightarrow \mathcal{P} \left( \mathbb{R}^d \right)$

Abstraction

Conclusion
\[ f : \mathbb{R}^0 \rightarrow \mathcal{P}(\mathbb{R}^0) \]
\[ f(x) = \{1\} \]
Linear Hybrid Automata

- simple continuous dynamics: conjunctions of linear constraints $a \cdot \dot{x} \leq b$, $a \in \mathbb{Z}^n$, $b \in \mathbb{Z}$
- All sets defined by Boolean combinations of linear constraints
\[ f(x) = \mathcal{P} \]

**Linear Hybrid Automata**

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**Post**: letting time elapse
### Linear Hybrid Automata

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**Post$_{c}$**: letting time elapse
**Post$_{d}$**: discrete transition
\[ f(x) = P \]

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$\text{Post}_c$: letting time elapse
$\text{Post}_d$: discrete transition
More expressive than LHA: \( f(x) = 0\{x\} \oplus \mathcal{P} \)

- Continuous dynamics: \( \dot{x} \in A_q\{x\} \oplus \mathcal{U}_q \)
- Switching hyperplanes or Polyhedral guards.
Reachability for LTI:

- Time discretization: \( \dot{x} \in A\{x\} \oplus U \rightarrow x_{k+1} \in \Phi\{x_k\} \oplus V \)
- Computation of the \( N \) first terms of:

\[
\Omega_{n+1} = \Phi \Omega_{n+1} \oplus V
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\( \Omega_{n-1} \) may have more than \( \frac{(2n)^{d-1}}{\sqrt{d}} \) vertices.
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Reachability for LTI:

**Computation of the first terms of:**

\[ \Omega_{n+1} = \alpha(\delta(\Omega_n) \oplus V) \]

- Time discretization: \( x \in A\{x\} \oplus U \rightarrow x_{k+1} \in \Phi\{x_k\} \oplus V \)

\[ f(x) = A\{x\} \oplus U \]

### Table

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- Time discretization: \( \dot{x} \in A\{x\} \oplus \mathcal{U} \rightarrow x_{k+1} \in \Phi\{x_k\} \oplus \mathcal{V} \)
- Computation of the first \( N \) terms of:

\[
\overline{\Omega}_{n+1} = \alpha(\Phi \gamma(\overline{\Omega}_{n+1}) \oplus \mathcal{V})
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Reachability for LTI:

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The approximation error can be exponential in the number of steps! \( \rightarrow \) wrapping effect
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\[
T : \mathcal{X} \mapsto \Phi \mathcal{X} \oplus V
\]

\[
(\alpha \circ T \circ \gamma)^n = \alpha \circ T^n \circ \gamma
\]

\[
(\alpha \circ T \circ \gamma) \circ \alpha = \alpha \circ T
\]
\[ f(x) = A\{x\} \oplus \mathcal{U} \]

\[ \Omega_n = \Phi^n \Omega_0 \oplus \bigoplus_{i=0}^{n-1} \Phi^i \mathcal{V} \]
\[ f(x) = A\{x\} \oplus U \]

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\[ A_0 = \Omega_0 \quad \quad A_{n+1} = \Phi A_n \]

\[ \nu_0 = \nu \quad \quad \nu_{n+1} = \Phi \nu_n \]

\[ S_0 = \{0\} \quad \quad S_{n+1} = S_n \oplus \nu_n \]

Then: \[ \Omega_n = A_n \oplus S_n \]

- \( A_i \) and \( \nu_i \) have a constant representation size.
- We can exploit redundancies of \( S_i \) (zonotopes, support functions).
\[ f(x) = A\{x\} \oplus U \]

\[ \Omega_n = \Phi^n \Omega_0 \oplus \bigoplus_{i=0}^{n-1} \Phi^i \nu \]

\[ A_0 = \Omega_0 \quad A_{n+1} = \Phi A_n \]
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Approximations can still be interesting:

- We are only interested in one individual \( \Omega_i \).
- We want to use a tool that can not exploit the redundancies.
\[ f(x) = A\{x\} \oplus \mathcal{U} \]

\[ \Omega_n = \Phi^n \Omega_0 \oplus \bigoplus_{i=0}^{n-1} \Phi^i \mathcal{V} \]

\[ \mathcal{A}_0 = \Omega_0 \]
\[ \mathcal{A}_{n+1} = \Phi \mathcal{A}_n \]
\[ \mathcal{V}_0 = \mathcal{V} \]
\[ \mathcal{V}_{n+1} = \Phi \mathcal{V}_n \]
\[ \overline{S}_0 = \{0\} \]
\[ \overline{S}_{n+1} = \alpha(\gamma(\overline{S}_n) \oplus \mathcal{V}_n) \]

Approximations can still be interesting:
- We are only interested in one individual \( \Omega_i \).
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\[ T : (\mathcal{X}, \mathcal{Y}, \mathcal{Z}) \mapsto (\Phi \mathcal{X}, \Phi \mathcal{Y}, \mathcal{Z} \oplus \mathcal{Y}) \]
\[ (\alpha \circ T \circ \gamma)^n = \alpha \circ T^n \circ \gamma \]
\[ \alpha(\gamma(\alpha(\mathcal{Z})) \oplus \mathcal{Y}) = \alpha(\mathcal{Z} \oplus \mathcal{Y}) \]
$f(x) = A\{x\} \oplus \mathcal{U}$
\[ f(x) = A\{x\} \oplus \mathcal{U} \]
\[ f(x) = \{ Ax \mid A \in \mathcal{A} \} \]

More expressive than \( f(x) = A\{x\} \oplus \mathcal{U} \) (in smaller dimension):

\[ f(x) = \left\{ \begin{pmatrix} A & u \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ 1 \end{pmatrix} \mid u \in \mathcal{U} \right\} \]

- Time discretization: \( \dot{x} \in \mathcal{A}x \longrightarrow x_{n+1} \in \mathcal{M}x_n \)
- Use of set representations in the space of Matrices.
\[ f(x) = \{ Ax \mid A \in \mathcal{A} \} \]

- 8 variables
- 3 discrete locations
\[ f(x) = \{Ax \mid A \in \mathcal{A}\} \]

- 8 variables
- 3 discrete locations
If we want to use similar techniques:

- Adapt integration schemes:

\[ \mathcal{X} \mapsto \mathcal{X} \oplus \delta f(\mathcal{X}) \oplus \mathcal{E} \]

- Abstract
Abstraction

\[ f : \mathbb{R}^0 \rightarrow \mathcal{P}(\mathbb{R}^0) \]

\[ f(x) = \{1\} \]
\[ \tilde{f}(x) = \mathcal{P} \]
\[ \hat{f}(x) = A\{x\} \oplus \mathcal{U} \]
\[ \bar{f}(x) = A\{x\} \]
Rectangular partition.

We need to know if $f_i(G) \cap \mathbb{R}^+$ is empty.
\( \bar{f} : \mathbb{R}^0 \rightarrow \mathcal{P}(\mathbb{R}^0) \)

Smooth partition.

- Sign conditions on a set of functions and their derivatives.
- No transition from \((x > 0, \dot{x} > 0)\) to \((x < 0, \dot{x} > 0)\)

We need to check emptiness of the cells.
\[ \bar{f}(x) = \{1\} \]

Timed automata.

- Partition of the state space in slices
- Clocks measure time to get from one slice to the other
- We need to know upper and lower bounds for \( f_i(S) \).
- Easier when Lyapunov functions are availables
\[ \bar{f}(x) = P \]

**LHA**

- Polyhedral partition
- For each cell \( C \) of the partition, we need to know \( f(C) \)
\[ \tilde{f}(x) = \mathcal{P} \]

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![Diagram showing LHA]
\( f(x) = \mathcal{P} \)

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![Diagram showing Polyhedral partition with initial states, final states, and reachable final states.](image)
\[ \tilde{f}(x) = \mathcal{P} \]

- **LHA**
  - Polyhedral partition
  - For each cell \( C \) of the partition, we need to know \( f(C) \)

\[ \begin{align*}
\tilde{f}(x) &= \{1\} \\
\tilde{f}(x) &= \mathcal{P} \\
\tilde{f}(x) &= A\{x\} \oplus U \\
\tilde{f}(x) &= A\{x\}
\end{align*} \]

- Introduction
- State of the Art
- Abstraction
  \( \bar{f} : \mathbb{R}^0 \rightarrow \mathcal{P}(\mathbb{R}^0) \)
  \( \bar{f}(x) = \{1\} \)
  \( \bar{f}(x) = \mathcal{P} \)
  \( \bar{f}(x) = A\{x\} \oplus U \)
  \( \bar{f}(x) = A\{x\} \)
- Conclusion

Colas Le Guernic
\[
\bar{f}(x) = A\{x\} \oplus U
\]

For each cell \( C \) of the partition:

- Choose linearization \( A \)
- Compute \( U = \{ y - Ax \mid x \in C, y \in f(x) \} \)

We want \( U \) to be as small as possible, how do we choose \( A \)?
\[ \bar{f}(x) = A\{x\} \oplus \mathcal{U} \]

For each cell \( \mathcal{C} \) of the partition:

- Choose linearization \( A \)
- Compute \( \mathcal{U} = \{y - Ax \mid x \in \mathcal{C}, y \in f(x)\} \)

We want \( \mathcal{U} \) to be as small as possible, how do we choose \( A \)?

We do not really know...
\( \bar{f}(x) = A\{x\} \oplus U \)

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We do not really know...

One guess is to take the Jacobian at the center of the cell.
\[ f(x) = \{ Ax \mid A \in A \} \]
\( \tilde{f}(x) = \{Ax \mid A \in \mathcal{A}\} \)

One guess is to take the Jacobians at every point of the cell.
\[ \tilde{f}(x) = \{ Ax \mid A \in A \} \]

One guess is to take the Jacobians at every point of the cell. If we find a subset of variables such that:

- \( f \) is linear in these variables
- no product of two of these variables appear in \( f \)

We do not need to partition along these variables.
Choosing the right abstraction is rarely easy.

- choice of the partition
- choice of the class of abstraction
- choice of the abstraction in this class
Choosing the right abstraction is rarely easy.

- choice of the partition
- choice of the class of abstraction
- choice of the abstraction in this class
- modifying the number of continuous variables
- combining different classes of abstractions
Thank you